Math Fundamentals PoW Packet

Something Fishy

Problem 4723 • https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in February 2008. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

Standards

In **Something Fishy** students are asked to find the maximum number of goldfish and koi to go in the fish pond. The **key concepts** are proportional reasoning, measurement, multiplication and division. It is appropriate for students to use calculators to carry out the computation involved. Students who lack skills in division, or the confidence to use them, may use a guess-and-check strategy that relies more on multiplication or repeated addition.

If your state has adopted the Common Core State Standards, this alignment might be helpful.

Grade 3: Operations & Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

Use the four operations with whole numbers to solve problems.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

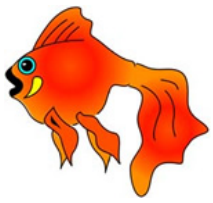
Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

The Problem

Something Fishy

The students at Highland Middle School are designing a schoolyard garden, to include a small pond. They have decided that they will build their pond to hold 1000 gallons of water. They have also selected two types of fish to occupy their pond: goldfish and koi. The students want to have an equal number of goldfish and koi.



Goldfish can grow up to 12 inches in length, while a koi can reach a length of 24 inches. Both types of fish require two gallons of water per inch of fish length.

What is the maximum number of each breed they can purchase, making sure they have enough space in the water for each fish?

Extra: They decide to make the pond 4 feet deep to discourage raccoons from eating the fish. Find the approximate area of the pond's surface. [There are about 7.5 US gallons of water in a cubic foot.]

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The students should put no more than 13 of each kind of fish in the pond.

If your answer **doesn't** match ours,

- did you remember they want equal numbers of goldfish and koi?
- did you remember that each fish needs 2 gallons on water **per inch** of length?
- did you think about the best way to round your calculations?
- did you check your arithmetic?

If you used guess-and-check, did you tell . . .

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Guess and Check

After reading the problem, our group decided to write down everything we noticed.

Students at Highland Middle School are building a pond
The pond will contain 1000 gallons of water
There will be two types of fish in the pond: goldfish and koi
There will be an equal number of goldfish and koi
Goldfish can grow to be 12 inches long
Koi can grow to be 24 inches long
Both fish require 2 gallons of water per inch of fish length

We saw that if each goldfish needs 2 gallons of water for every inch of length, then one goldfish could occupy up to 24 gallons, since 12 • 2 = 24 gallons. Similarly, each koi needs 2 gallons of water per inch, so one koi could occupy up to 48 gallons of water, since 24 • 2 = 48 gallons.

We made a table and tested different numbers of fish to see how many the students could fit in their pond. We made sure to keep the number of goldfish and koi equal.

Number of Fish	Gallons of water goldfish need	Gallons of water koi need	Total gallons of water
5	24 • 5 = 120	48 • 5 = 240	120 + 240 = 360
10	24 • 10 = 240	48 • 10 = 480	240 + 480 = 720
15	24 • 15 = 360	48 • 15 = 720	360 + 720 = 1080
14	24 • 14 = 336	48 • 14 = 672	336 + 672 = 1008
13	24 • 13 = 312	48 • 13 = 624	312 + 624 = 936

We saw that using 14 fish would be a little too much over 1000 gallons, so the students can use 13 of each type of fish in their pond!

Method 2: Find Water Required for a Pair

Each fish requires 2 gallons of water per inch of fish length. We noticed that students will need to allow 24 gallons of water for each 12-inch adult goldfish, since 12 • 2 = 24. They will also need to allow 48 gallons of water for each 24-inch adult koi, since 24 • 2 = 48. They also want an equal number of goldfish and koi.

One goldfish and one koi together, or one pair, will require 72 gallons of water since

24 gal + 48 gal = 72 gal

To find out how many pairs of fish can fit in the pond, we divided 1000 gallons of water by 72 gallons of water per pair. 1000 gal ÷ 72 gal per pair = 13.888… pairs. In this case rounding up would result in more fish than the pond could support, so we rounded down and got 13. The students should put no more than 13 of each kind of fish in the pond.

Method 3: Proportional Reasoning to Find Water Required

Our group noticed that koi grow to be twice the length of goldfish and that both breeds require 2 gallons of water per inch of length. Knowing this we realized that the goldfish will need 1/3 of the water and the koi will need 2/3. We found out what portion of the 1000 gallons of water in the pond will be occupied by each breed of fish:

1000 ÷ 3 = 333.33… gal for goldfish
12 in • 2 gal = 24 gal of water per goldfish
333.33 gal ÷ 24 gal per fish = 13.88… goldfish

According to our calculations, the students can put 13 goldfish in the pond. This means that they should also be able to put 13 koi in the pond. We decided to check our work to make sure we were right by seeing how many total gallons of water 13 goldfish and 13 koi would need.

13 goldfish • 24 gal per fish = 312 gal of water needed
13 koi • 48 gal per fish = 624 gal of water needed
312 gal + 624 gal = 936 gal

One more goldfish and koi would require 24 + 48, or 72 gallons of water, which would be more than 1000 gallons. We finally concluded that the students could have 13 of each breed of fish in their pond!

Method 4: Algebra

We knew from the problem that each type of fish requires 2 gallons of water per inch of length in the school's new 1000 gallon pond. A 12-inch goldfish can occupy up to 24 gallons of water since 12 • 2 = 24. A 24-inch koi can occupy up to 48 gallons of water since 24 • 2 = 48. Using this information, we wrote an algebraic equation to figure out the total amount of each type of fish the students can fit in their pond. We let x equal the number of each breed of fish they can put in the pond.

24x + 48x = 1000
72x = 10000
x = 1000 ÷ 72
x = 13.89

We knew that we could not have 0.89 of a fish, so we figured out that there can be 13 of each type of fish in the pond.

Extra: We knew that there are 7.5 gallons in 1 cubic foot. We set up a proportion to find how many cubic feet there are in 1000 gallons.

$$\frac{1 \text{cubic ft.}}{7.5 \text{ gallons}} = \frac{?\text{cubic ft.}}{1000 \text{ gallons}}$$

$$7.5 \bullet ? = 1000$$

$$? = 1000 \div 7.5$$

$$? = 133.33 \text{ cubic ft.}$$

There are 133.33 cubic feet in 1000 gallons of water. This is the volume of the entire pond. The student's made a pond 4 feet deep, so to find the area of the pond's surface we divided the volume by 4.

133.33 ÷ 4 cu ft = 33.3 sq ft

The area of the pond's surface is approximately 33.3 sq ft.

Teaching Suggestions

In strategies involving division, students are likely to arrive at answers with decimal remainders greater than 0.5. This problem provides an opportunity to discuss the need to interpret the remainder in a way that makes sense in the context of the problem. Rounding up in this problem results in more fish than 1000 gallons can support.

Students who use a guess-and-check approach should be encouraged to record their trials and to test new numbers based on what they learned from prior tests. Taking advantage of observed patterns helps them arrive at correct answers more efficiently.

Sample Student Solutions

focus on **Interpretation**

In the solutions below, we've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. Our comments focus on what we feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands • that the problem asks to find the maximum number of koi and also goldfish to go in the pond. • that the number of goldfish and koi must be equal. • that each goldfish needs 24 gallons of water and each koi needs 48 gallons of water. • how to interpret remainders appropriately.	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

Michael
age 10

Interpretation
Novice

I think it would be 36,000gallons.

you x 1000 by 12then1000 x12000 then add12000 +24000=36000

Michael seems unaware of what the problem is asking. We'd start by having him paraphrase the problem and list the things he notices and wonders.

Pookie
age 11

Interpretation
Apprentice

There answer is 480 Goldfish, and 480 Koi

I realized that to have an even number of goldfish and koi, you must cut the 1000 in half. I also knew that since the fish must have two gallons for every inch, the length needed to be doubled. So I did 24 in 500, and 48 in 500 and one less was 480 on both of them.

Pookie understands that the number of goldfish and koi must be equal, and also that 24 and 48 have a role in the problem. I'd ask what those numbers represent, and then ask her/him to find how much water 480 goldfish would require.

Daniel
age 12

Interpretation
Apprentice

There are 27 goldfish and koi in the pond.

I used the Guess & Check method.
I tried 25, which came to 900 gallons.
I added 1, (26 total) which totals 936 gallons.
I added one again, which is 27. (Total" 972 gallons)
You can't add another one because the total would come to 1008.
The total is 27.

*Since Daniel seems to be using increments of 36 (12 + 24), it appears that he has missed the fact that each fish requires "**two** gallons of water per inch of fish length." Once he understood that, I'd ask for more detail about his testing.*

Hovey
age 12

Interpretation
Apprentice

14 goldfish & 14 kois

I first took the sum of the lenthes of the two kinds of fish, 12+24=36 ~ which requires 72 gal. of water. Then I divided 1000 gallons of water out of 72 gallons.

1000/2=13.89=14

That means there are fourteen goldfish and kois.

Hovey understands the key ideas and uses a sound strategy. He has been taught to round up when remainders are >0.5. I'd ask him to find how much water 14 fish would need.

Patrick
age 11

Interpretation
Practitioner

The maximum number of each breed one can purchase is 13.

I used the Guess and Check strategy to find the solution. First, I made a chart with two columns with Goldfish at the top of one and Koi at the top of the other. I guessed 5. I had to multiply the 5 by 12 and 24 to find out the number of inches of fish involved. Then I had to multiply those numbers by 2 to get the number of gallons of water needed. Then I added the two gallon amounts together. The answer was too small. Then I tried 10 and it was too short. Then I tried 15 and it was too much. When I tried 13, it was not enough, but when I tried 14, it was too much. So it had to have been 13.

Patrick demonstrates good understanding and writes a solid explanation of his steps. I'd ask him to include his chart and ask him to observe patterns or relationships that might lead to a more direct solution path.

Takuma
age 11

Interpretation
Practitioner

There are 13 fishes of each type.

First, I added 12 inches which is the length of the goldfish to 24 inches which is the length of the koi, 12+24=36 inches.
Next, I multiplied 36 by 2 because per inch it will require 2 gallons.
36x2=72 gallons.
Then, I divided 72 into 1000 which is 13.88...
Finally , I will multiply to check:
72x13=936<1000 OK
72x14=1,008>1000 X
And my answer is 13 of each fish.

Takuma demonstrates good understanding of the problem and uses an efficient, direct approach. I like the fact that Takuma showed that the answer met the conditions of the problem. I would ask what 13.88 represents and how the remainder was interpreted.

Justin
age 11

Interpretation
Expert

The maximum number of each breed they can purchase, making sure they have enough space in the water for each fish is 13.

Extra - The approximate area of the pond's surface is 33 1/3 Square Feet.

Answer:
1) 1000 Gallons of Water/2 = 500 Gallons per inch
2) 500/36 total inches for the two breeds = 13.88
3) So maximum of each breed is 13.

Extra:
1) 1000/7.5 = 133.33 Cubic Feet.
2) L x W x D = 133.33 Cubic Feet
3) L x W x 4 = 133.33 Cubic Feet
4) 133.33/4 = 33.33
5) L x W = 33.33
6) 33 1/3 Square Feet

Justin's explanation could be clearer and more complete with some detail justifying his steps, but he clearly understands the problem and has demonstrated two interesting approaches.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

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